Final report for AFSOR Grant Number F49620–96–1–0190
Coupled Elastic Surface Waves in Curved Structures

John G. Harris, Theoretical and Applied Mechanics, UIUC
216 Talbot Laboratory, 104 South Wright Street
Urbana, IL 61801

Summary

A mathematical description of coupled, surface-wave propagation in elastic waveguides whose properties change slowly in the lateral and/or transverse directions has been constructed. This construction includes curved waveguides (shells) whose radius of curvature is large with respect to thickness. The work was undertaken to explore a nondestructive way to interrogate an inaccessible side of a structure by launching a surface wave on the other, accessible side. For flat waveguides (plates) whose thickness varies slowly in the propagation direction the surface waves couple very efficiently provided the thickness is on the order of a wavelength, but as the waveguide grows in thickness the coupling is quickly lost. The coupling in a curved waveguide occurs provided the ratio of thickness to radius of curvature is of the order of 1/20 and the thickness is on the order of a wavelength. The modifications caused by the varying geometry enter the form of the guided wave as slowly varying complex amplitudes and do not substantially affect the dispersion relation, which remains very close to that of a flat plate. Measurements to verify the existence of the coupling between surfaces by a surface wave were made in almost flat, metal plates. The coupling was found by Fourier transforming the measurements of surface displacement to find the dominant frequency, which represented the inverse of the coupling distance. Further development is needed to transform this work into a reliable nondestructive evaluation technique.

Objectives of the work

1. Develop an asymptotic theory for time-harmonic disturbances that describes the propagation of the two lowest waveguide modes in a curved elastic structure whose radii of curvature are large and vary slowly, and explore how similar they are to those of a flat plate.

2. Explore under what conditions the propagation of the two lowest waveguide modes of a structure can be described as the propagation of a Rayleigh surface wave along one or the other of the two bounding surfaces and explore the coupling that takes place between the surfaces.

20000308 008

REPORT DOCUMENTATION PAGE

The public reporting burden for this collection of information is estimated to average 1 hour per response, including th. gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this... of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for information. (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person... subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.
PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
11/02/00	Final Report	May 1996 – November 1999

4. TITLE AND SUBTITLE

Coupled Elastic Surface Wave in Curved Structures

5a. CONTRACT NUMBER

5b. GRANT NUMBER

F49620-96-1-0190

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

John G. Harris

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Theoretical and Applied Mechanics
University of Illinois
216 Talbot Lab., 104 S. Wright Street
Urbana, IL 61801

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

AFOSR/NA
801 N. Randolph St.
Arlington, VA 22203-1977

10. SPONSOR/MONITOR'S ACRONYM(S)

AFOSR

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

A mathematical description of coupled, surface-wave propagation in elastic waveguides whose properties change slowly in the lateral or transverse directions has been constructed. This construction includes curved waveguides (shells) whose radius of curvature is large with respect to thickness. The work was undertaken to explore a nondestructive way to interrogate an inaccessible side of a structure by launching a surface wave on the other, accessible side. For flat waveguides (plates) whose thickness varies slowly in the propagation direction the surface waves couple very efficiently provided the thickness is on the order of a wavelength, but as the waveguide grows in thickness the coupling is quickly lost. The coupling in a curved waveguide occurs provided the ratio thickness to radius of curvature is of the order of 1/20 and the thickness is on the order of a wavelength.

15. SUBJECT TERMS

Coupled Rayleigh waves, nondestructive evaluation, ultrasonics, microwave acoustics

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE	UU	7	John G. Harris
unclassified....	..				19b. TELEPHONE NUMBER *(Include area code)* 217-333-7433

3. Show that using the coupled elastic surface waves is one way to inspect the inner, inaccessible surface of a structure for damage from the outer accessible surface.

Relevance to the Air Force

The increasingly longer service life of Air Force and civilian structures means that they must be continuously monitored for damage to ensure their reliability. A surface wave, in contrast to a bulk wave, coupling to the interior surface of a plate or shell would strike a small surface-breaking crack broadside or would remain in continuous contact with a patch of corrosion and hence be strongly perturbed. This would indicate the presence of such damage at an early stage before it became severe, and hence allow the structure to be modified or removed from service before it failed.

Outcomes

Variable thickness waveguide. At high frequencies it is almost impossible to excite either of the two lowest Rayleigh-Lamb modes in an elastic plate separately. Instead both modes are excited and, because the lowest is antisymmetric and the next lowest symmetric, the two propagate together and mimic a Rayleigh surface wave that couples back and forth between one surface and the other. We measured this phenomena to check that it did indeed exist. This measuremnt work is reported in Ti *et al* (1997). We were successful in finding the phenomena though its measuremnt was not as direct as we had at first thought it would be. A measurement of the particle displacement suggested the coupling phenomena but was not conclusive. However, Fourier trasforming the record of the particle displacement clearly indicated the coupling from one surface to the other. That is, there appeared a dominant frequency in the measured data that indicated a steady rise and fall in the particle displacement and thus indicated the coupling. This dominant frequency agreed with that predicted by the theory, though it did not appear as a sudden sharp peak but as a smooth hump indicating that the coupling was not as clean as the theory would suggest. Nevertheless, *this suggests how a nondestructive testing measurement should be made: one should look at the measurements in the frequency domain, rather than in the spatial one.*

We undertook to calculate this coupling phenomena in plates with variable thickness. Figure 1 shows the problem of interest, while the method of approach is outlined in Figure 2. The asymptotic expansions used have the form

(1)
$$u_i = e^{i\theta(x)/\delta} \sum_{\nu \geq 0} u_{i\nu} (\delta)^\nu$$

$$\tau_i = e^{i\theta(x)/\delta} \sum_{\nu \geq 0} \tau_{i\nu} (\delta)^\nu.$$

wedge transducer phased
matched to Rayleigh wave

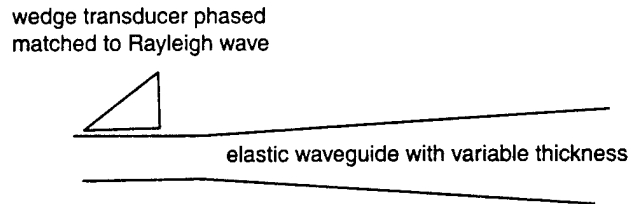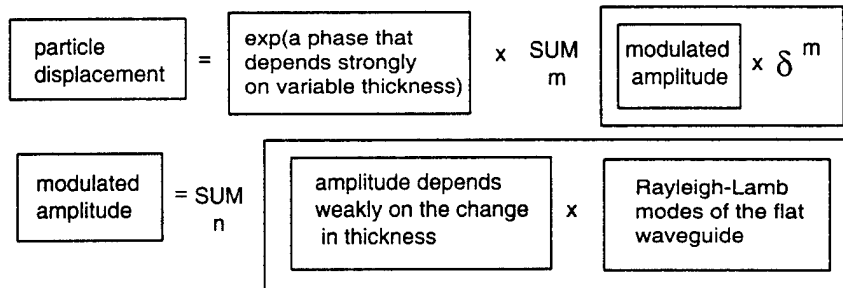elastic waveguide with variable thickness

Figure 1: A wedge-transducer excites the two lowest Rayleigh-Lamb modes in an elastic plate whose thickness varies slowly with respect to wavelength.

| particle displacement | = | exp(a phase that depends strongly on variable thickness) | x | SUM m | modulated amplitude | x δ^m |

| modulated amplitude | = SUM n | amplitude depends weakly on the change in thickness | x | Rayleigh-Lamb modes of the flat waveguide |

$\delta \ll 1$ is the change in thickness over a wavelength

Figure 2: A schematic outline of the mathematical approach taken. The term exp means exponential and contains the rapidly oscillating part of the wave. The modulated amplitude is expanded in Rayleigh-Lamb modes at each point along the waveguide. The modes are those corresponding to the thickness at that point.
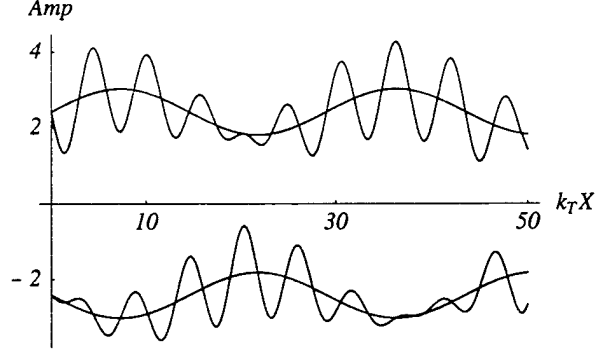
Figure 3: The normal particle displacement is shown for a sinusoidally varying waveguide. The magnitude of the displacement is exaggerated. The frequency of the thickness variation is very much smaller than the frequency of the elastic waves.

Setting the coefficient of each power of δ to zero gives the recursive system

$$(2) \qquad [\mathcal{L} - id_x\theta(x)] \begin{pmatrix} u_{1\nu} \\ u_{3\nu} \\ \tau_{1\nu} \\ \tau_{3\nu} \end{pmatrix} = \partial_x \begin{pmatrix} u_{1(\nu-1)} \\ u_{3(\nu-1)} \\ \tau_{1(\nu-1)} \\ \tau_{3(\nu-1)} \end{pmatrix}.$$

The leading equation of the recursive system yields the following eigenvalue problem.

$$(3) \qquad \mathcal{L} \begin{pmatrix} u_{10} \\ u_{30} \\ \tau_{10} \\ \tau_{30} \end{pmatrix} = i\beta(x) \begin{pmatrix} u_{10} \\ u_{30} \\ \tau_{10} \\ \tau_{30} \end{pmatrix}$$

with
$$(4) \qquad d_x\theta(x) = \beta(x).$$

The operator $\mathcal{L}$ is given by

$$(5) \qquad \mathcal{L} = \begin{pmatrix} 0 & -a\partial_z & b & 0 \\ -\partial_z & 0 & 0 & 1 \\ -1 & 0 & 0 & -\partial_z \\ 0 & -c\partial_z^2 - 1 & -a\partial_z & 0 \end{pmatrix}.$$

This formulation of the eigenvalue problem for a Rayleigh-Lamb modes is not widely known or used. The τ_i are the components of traction and the u_i those of displacement. The a, b and c are combinations of elastic constants. z is the transverse coordinate; x the lateral one. This formulation can be extended to solve a very broad class of elastic waveguide problems. The details of this calculation are given in the two papers Folguera and Harris (1999a,b).

4

caustic

provided the caustic remains outside the upper surface
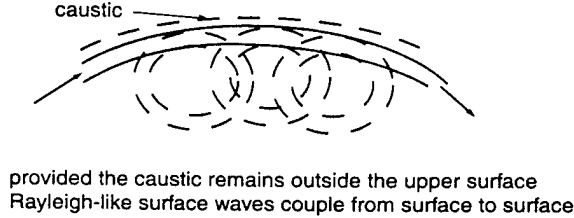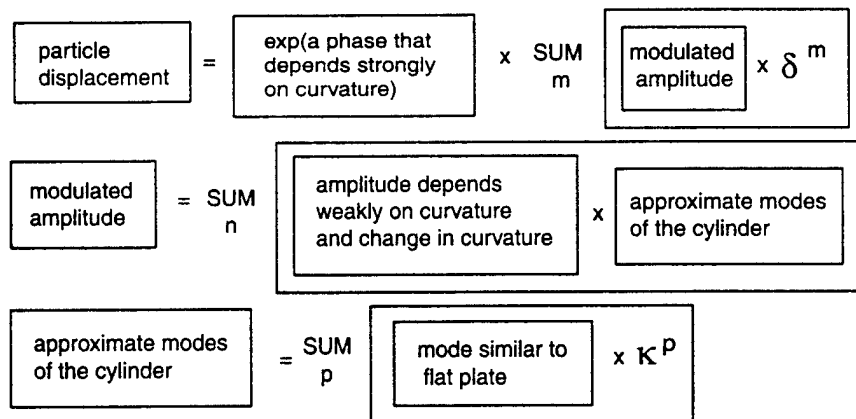Rayleigh-like surface waves couple from surface to surface

Figure 4: The modes of a cylinder are used to approximate those of a shell with a more general curvature.

Figure 3 shows our results for a plate whose surfaces vary sinusoidally. The figure exaggerates the particle displacement, which is $\Re(u_3)$, the real part of the particle displaceent normal to the surface.

Curved waveguides. I have been calculating the propagation of surface waves on an arbitrarily curved shell-like structure by trying to imitate the calculation just described. My first attack on the inplane elastic problem for the curved structure proved unsatisfactory. I backed away, attacked an analogous scalar problem, solved that quite satisfactorily and returned to the inplane elastic problem. This new calculation is now almost complete, though no numerical calculations of results have been done. Figure 4 represents geometrically the general scheme of the approximation, while Fig. 5 tries to capture the mathematical construction. There are two small parameters in the problem, namely, κ, the curvature scaled with the wavelength and δ the rate of change of the curvature over a wavelength. Approximating in δ leads to a recursive sytem of equations similar to (2) and an eigenvalue problem for the thickness similar to (3). The eigenvalue problem requires finding the transverse eigenmodes for the corresponding cylindrical structure (Fig. 4), which leads to combinations of Hankel functions. Rather than proceed in this way, the eigenvalue problem is attacked directly using an expansion in κ. It emerges from the analysis that the product of thickness times the curvature $h\kappa$ becomes an important parameter. Two limiting cases of the eigenvalue problem are being worked out, namely, $h\kappa \ll 1$ and $h\kappa \gg 1$

The principal conclusions that I have reached so far are the following.

1. The presence of any curvature affects the mode shape and modulated amplitude more than it does the dispersion relation. The dispersion relation is almost identical to that of a flat plate for large (that is, several shear wavelengths) radii of curvature.

particle displacement = exp(a phase that depends strongly on curvature) × SUM over m [modulated amplitude] × δ^m

modulated amplitude = SUM over n [amplitude depends weakly on curvature and change in curvature] × [approximate modes of the cylinder]

approximate modes of the cylinder = SUM over p [mode similar to flat plate] × κ^p

$\delta \ll 1$ curvature varies slowly over a wavelength

$\kappa \ll 1$ curvature itself is small with respet to wavelength

Figure 5: A schematic representation of the structure of the expressions. The two small parameters are δ, the rate of change of the curvature with wavelength, and κ, the curvature relative to the reciprocal wavelength. Note that in contrast to Fig.2 an additional expansion is neededto solve the eigenvalue problem.

2. In any curved structure, the guided waves form a caustic (each guided mode can be represented by a set or pencil of rays; the rays form an envelope called a caustic). In the scalar problem the caustic can never be forced to lie beyond the outer concave surface[1], though it can be forced to lie outside the inner convex surface .

3. For a certain range of the parameters the caustic, for the inplane elastic problem, can be forced out of the guide into a region beyond the outer concave surface, as shown in Fig. 4. In this case coupling of Rayleigh surfaces waves between the surfaces occurs more or less as it does in the flat plate, provided $h\kappa \ll 0.1$.

In short, there is a window, largely defined by the frequency, shear-wave speed, curvature and thickness in which the coupling of a surface wave from the inner to the outer curved surface occurs. The main task remaining is to define that window.

Transitions to industry / Future

I have become involved with the Technical Center, the Advanced Materials Group, Caterpillar, Inc. The mathematical approach developed for the curved waveguide work is now being used to study propagation in thermal barrier coatings. These coatings are used in engine components to allow the engine to

---

[1] Concave and convex are determined by imagining yourself within the guide

be operated at higher temperatures. The guided waves will be used to assess the consistency in growing these coatings and to monitor the their integrity when the engine is in service. In the future (next two years or so) I intend to continue to study propagation in coatings. Please do not hesitate to contact me if you wish to learn more about this work.

Administration of the grant

Starting in the fall of 1997, Dr. Alejandra Folguera was supported fully on the present grant as a post-doctoral associate. Starting in September 1998, Dr. Folguera was supported one-third time on this grant, while teaching in the Mathematics Department, UIUC, for the other two-thirds. In May 1999 she choose to end her appointment. With what funds remained I supported myself over the summer of 1999. Two publications acknowledging this grant have appeared, namely Folguera and Harris (1999a,b). A further paper describing the curved surface work will be submitted for publication and a shorter paper descibing this work will appear in a conference proceeding.

References

Folguera, A. and Harris, J.G. 1999a. Coupled Rayleigh surface waves in a slowly varied waveguide. *Proc. R. Soc. Lond.* A **455**: 917–931.

Folguera, A. and Harris, J.G. 1999b. Propagation in a slowly varying elastic waveguide. In *Mathematical and Numerical Aspects of Wave Propagation*, ed. J.A. DeSanto, pp. 434–436. Philadelphia: SIAM. Ti, B.W., O'Brien, W.D. and Harris, J.G. 1987. measuremnets of coupled Rayleigh wave propagation in an elastic plate. *J. Acoust. Soc. Am.* **102**: 1528–1531.